1. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

a) $\underline{a} = \frac{d\underline{v}}{dt}$ $\qquad \underline{v} = 3t^{1/2}\underline{i} - 2t\underline{j}$

$\underline{a} = \frac{3}{2}t^{-1/2}\underline{i} - 2\underline{j}$ ① ①

b) $\underline{v} = k(\underline{i} - \underline{j})$

$\underline{v} = \begin{pmatrix} k \\ -k \end{pmatrix} = \begin{pmatrix} 3\sqrt{t} \\ -2t \end{pmatrix}$

comparing elements: $k = 3\sqrt{t}$

$-k = -2t$

$\Rightarrow 3\sqrt{t} = 2t$ ①

$0 = \sqrt{t}(2\sqrt{t} - 3)$ ①

$\therefore t = 0$ or $\sqrt{t} = \frac{3}{2} \Rightarrow t = \frac{9}{4}$ ①

ignore

c) $\underline{r} = \int \underline{v}\,dt$ $\qquad \underline{v} = 3t^{1/2}\underline{i} - 2t\underline{j}$

$\underline{r} = 2t^{3/2}\underline{i} - t^2\underline{j} + \underline{c}$ ① ①

Question 1 continued

$\underline{r} = 2t^{3/2}\underline{i} - t^2\underline{j} + c$

when $t=1$, $\underline{r} = -\underline{j}$

$-\underline{j} = 2\underline{i} - \underline{j} + \underline{c}$

$0 = 2\underline{i} + \underline{c}$

$\underline{c} = -2\underline{i}$

$\underline{r} = (2t^{3/2} - 2)\underline{i} - t^2\underline{j}$ (1)

d) speed $= 10$

$\sqrt{(3\sqrt{t})^2 + (-2t)^2} = 10$ (1)

$9t + 4t^2 = 100$ (1)

$4t^2 + 9t - 100 = 0$

$(t-4)(4t+25) = 0$

$t = 4$ or $t = -6.25$

$t > 0 \therefore t = 4$ (1)

$\underline{r} = (2(4)^{3/2} - 2)\underline{i} - (4)^2\underline{j}$

$\underline{r} = 14\underline{i} - 16\underline{j}$ (1)

Distance $OP = \sqrt{14^2 + (-16)^2}$ (1)

$= 2\sqrt{113}$ m (1)

2. [*In this question, position vectors are given relative to a fixed origin.*]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

a) sub $t=2$ into $\underline{v}$: $\underline{v} = 3(2)^2\underline{i} - 6(2)^{1/2}\underline{j}$

$\underline{v} = 12\underline{i} - 6\sqrt{2}\underline{j}$

1 1

speed $= \sqrt{12^2 + (-6\sqrt{2})^2} = 6\sqrt{6} = 15\text{ ms}^{-1}$ (2sf)

1 1

b) $\underline{a} = \frac{d\underline{v}}{dt} = 6t\underline{i} - 3t^{-1/2}\underline{j}$

1

c) $\underline{r} = \int \underline{v}\,dt = t^3\underline{i} - 4t^{3/2}\underline{j} + \underline{c}$ 1

sub in $t=4$, $\underline{r} = \underline{i} - 4\underline{j}$

$\underline{i} - 4\underline{j} = (4)^3\underline{i} - 4(4)^{3/2}\underline{j} + \underline{c}$ 1

$\underline{i} - 4\underline{j} = 64\underline{i} - 32\underline{j} + \underline{c}$

$\underline{c} = -63\underline{i} + 28\underline{j}$

$\therefore\ \underline{r} = (t^3 - 63)\underline{i} + (-4t^{3/2} + 28)\underline{j}$

sub in $t=1$

$\underline{r} = -62\underline{i} + 24\underline{j}$ 1

3. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1} where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$ **(3)**

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$ **(2)**

(c) the acceleration of P at time t seconds **(2)**

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$ **(2)**

(a) $v = (0^2 - 3(0) + 7)i + (2(0)^2 - 3)j$

$v = 7i - 3j$ (1)

speed $= |v|$

$= \sqrt{7^2 + (-3)^2}$ (1)

$= \sqrt{58}$

$= 7.6 \text{ ms}^{-1}$ (1)

(b) $t^2 - 3t + 7 = 2t^2 - 3$ (1)

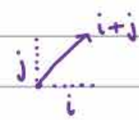
$t^2 + 3t - 10 = 0$

$(t+5)(t-2) = 0$

$\therefore t = -5$ or $t = 2$

$t = 2$ (1) because time can't be less than 0.

parallel to $(i+j)$ means coefficients of i and j are equal:



so $xi = xj$

(c) $\frac{dv}{dt}$ (1) $= (2t-3)i + (4t)j$

$\therefore a = (2t-3)i + (4t)j$ (1)

← acceleration is rate of change of speed over time, so find $\frac{dv}{dt}$

(d) $2t - 3 = 0$ (1)

$t = \frac{3}{2}$ seconds (1)

← 'perpendicular to i' means i-coefficient is 0.

so $0i + xj$

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

[*In this question,* **i** *is a unit vector due east and* **j** *is a unit vector due north. Position vectors are given relative to a fixed origin O.*]

At time t seconds, $t \geqslant 1$, the position vector of a particle P is **r** metres, where

$$\mathbf{r} = ct^{\frac{1}{2}}\mathbf{i} - \frac{3}{8}t^2\mathbf{j}$$

and c is a constant.

When $t = 4$, the bearing of P from O is 135°

(a) Show that $c = 3$ **(3)**

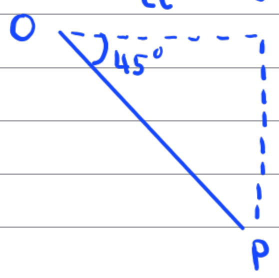
(b) Find the speed of P when $t = 4$ **(4)**

When $t = T$, P is accelerating in the direction of $(-\mathbf{i} - 27\mathbf{j})$.

(c) Find the value of T. **(4)**

a)

$ct^{1/2} = 2c$



$\frac{3}{8}t^2 = 6$ $\Rightarrow$ $2c\underline{i} - 6\underline{j}$ 1

when $t = 4$, $c(4)^{1/2} = 2c$

$\frac{3}{8}(4)^2 = 6$

using trigonometry : $\tan 45° = \frac{6}{2c}$ 1

$2c = 6$

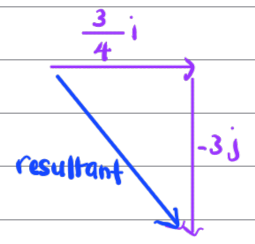
$c = 3$ (shown) 1

b) $r = 3t^{1/2}\underline{i} - \frac{3}{8}t^2\underline{j}$

speed, $v = \frac{dr}{dt}$ (1) $= \frac{3}{2}t^{-1/2}\underline{i} - \frac{3}{4}t\underline{j}$ (1)

when $t = 4$, $\frac{3}{2}(4)^{-1/2}\underline{i} - \frac{3}{4}(4)\underline{j}$

$= \frac{3}{4}\underline{i} - 3\underline{j}$ (1)



resultant $= \sqrt{(3/4)^2 + (-3)^2}$

$= \frac{\sqrt{153}}{4}$ (1)

c) acceleration, $a = \frac{dv}{dt}$ (1)

$= -\frac{3}{4}t^{-3/2}\underline{i} - \frac{3}{4}\underline{j}$ (1)

when $t = T$, acceleration of $P = (-\underline{i} - 27\underline{j})$

$$\frac{-\frac{3}{4}T^{-3/2}}{-\frac{3}{4}} = \frac{-1}{-27}$$ (1)

$$T^{-3/2} = \frac{1}{27}$$

$$\frac{1}{T^{3/2}} = \frac{1}{27}$$

$$T^{3/2} = 27$$

$T = 9$ (1)